T17 Concepts: Gibbs (1st + 2nd law) 16.Unified Fall 07

A) 1st law CV: $\frac{dE_{cv}}{dt} = \sum \dot{Q} + \sum \dot{W}_s + \sum \dot{m}(h + \frac{c^2}{2} + gz)$

for steady, no shaft work, adiabatic flow, single input-output:

$$0 = \cancel{\dot{m}}(h + \frac{c^2}{2} + gz)_{in} - \cancel{\dot{m}}(h + \frac{c^2}{2} + gz)_{out}$$

$\rightarrow h + \frac{c^2}{2} + gz = \text{const}$ or $dh + d(\frac{c^2}{2}) + gdz = 0$ (1)

1st and 2nd law combined $\rightarrow$ Gibbs: $Tds = dh - vdp$

Assume adiabatic reversible (isentropic) flow: $ds = 0$

Assume incompressible flow: $d\rho = 0$ or $\rho = \text{const}$

$\rightarrow dh = +\frac{1}{\rho}dp$ (2) combine (1) and (2)

$$\rightarrow \frac{1}{\rho}dp + d(\frac{c^2}{2}) + gdz = 0$$

OR $dp + \rho d(\frac{c^2}{2}) + \rho g dz = 0$ $\quad | \int$

OR $\underline{p + \rho \frac{c^2}{2} + \rho g z = \text{const}}$

Above is Bernoulli Equation - holds for steady, adiabatic, incompressible, inviscid (no friction $\rightarrow$ reversible) flow

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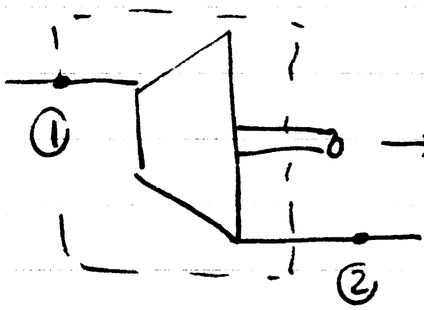
So, neglecting potential energy effects, $\underline{p_t = p + \frac{1}{2}\rho c^2 \text{ is constant}}$ along a streamline

B)



$P_1 = 6\,\text{bar}$, $T_1 = 870\,\text{K}$

$P_2 = 1\,\text{bar}$, $T_2 = 570\,\text{K}$

Concepts: 1st and 2nd law

i) 1st Law CV: $\dfrac{dE_{cv}}{dt} = \sum \dot{Q} + \sum \dot{W} + \sum \dot{m}\left(h + \frac{c^2}{2} + gz\right)$

neglect KE an PE effects, steady, adiabatic

$0 = -\dot{W}_s + \dot{m}(h_1 - h_2) \rightarrow w_s = c_p(T_1 - T_2)$

$\underline{w_s = 301.4\,\text{kJ/kg}}$

ii) 2nd law: $\Delta S_{total} \geq 0$?

$\Delta S_{total} = \Delta S_{1\to2}$ (no heat interaction with surroundings)

$\Delta S_{1\to2}$: Gibbs $\rightarrow T\,ds = dh - v\,dp$

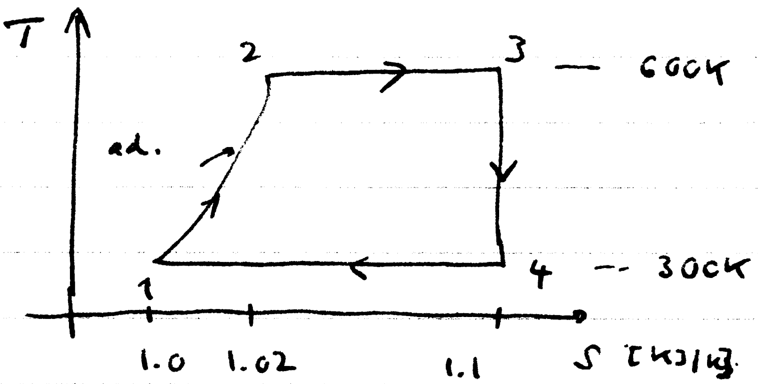
$$ds = c_p \frac{dT}{T} - R\frac{dp}{p}$$

$\Delta S_{1\to2} = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right) = \underline{89.5\,\text{J/kg-K}}$

$\Delta S_{1\to2} > 0$ so $\Delta S_{total} > 0 \rightarrow$ irreversible, non-ideal turbine

T18 Concepts: Gibbs, T-s diagrams 16. Unified Fall 07

Power cycle:



Processes:

1-2 adiab. irreversible compr. (lost work here!)

2-3 isothermal expansion

3-4 adiab. rev. (isentropic) expansion

4-1 isothermal compression

$$\eta_{th} = \frac{w}{q_A} = \frac{W}{Q_A} = 1 - \frac{Q_R}{Q_A}$$ per 1st Law for any cycle

rev. processes 2-3: $T\,dS = dQ \rightarrow \Delta S_{32} = \dfrac{Q_A}{T_2}$

$$Q_A = T_2 \Delta S_{32} = 600\,(1.1 - 1.02) = 48\text{ kJ}$$

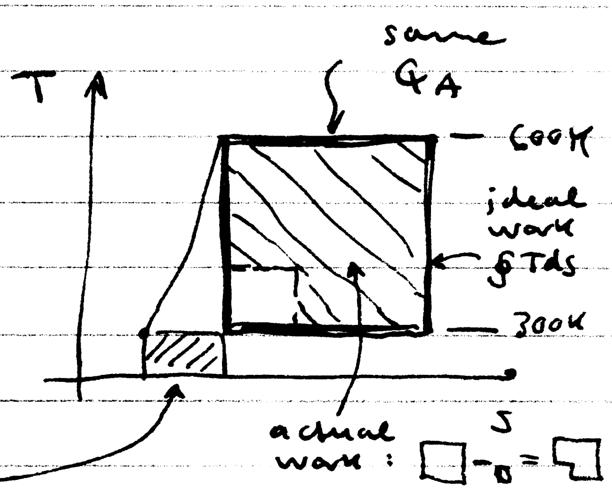
rev. process 4-1: $T\,dS = dQ \rightarrow \Delta S_{41} = -\dfrac{Q_R}{T_1}$

$$Q_R = -T_1 \Delta S_{41} = -300\,(1.0 - 1.1) = 30\text{ kJ}$$

$$\underline{\eta_{th} = 1 - \frac{T_1}{T_2} \cdot \frac{(S_4 - S_1)}{(S_3 - S_2)} = 0.375}$$

Carnot: $\underline{\eta_{th}^{c}} = 1 - \dfrac{T_1}{T_2} = \underline{0.5}$

$S_2 = S_1 \rightarrow$



Note: rejecting more heat in non-ideal cycle – difference in rejected heat is the <u>lost work</u>!